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Fundamental limits of wireless ad hoc networks: upper MO bounds

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Abstract: This report addresses the problem of deriving fundamental trade-off bounds for wireless ad hoc networks when multiple performance criteria are of interest. It proposes a MultiObjective (MO) performance evaluation framework composed of a broadcast and interference-limited network model, a steady state performance metric derivation inspired by a discrete Markov chain formalism and formulates the associated MO optimization problem. Pareto optimal performance bounds between end-to-end delay and energy for a capacity-achieving network are given for the 1-relay and 2-relay networks and assessed through simulations.

Key-words: fundamental limits, capacity, delay, energy, wireless networks, upper bound, multi-objective bound

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Evaluation de performance Pareto-optimale de réseaux sans-fil avec prise en compte de canaux à diffusion limités en interférence

Résumé : Ce rapport s'intéresse au problème du calcul des limites fondamentales d'un réseau sans-fil d'un point de vue multi-critères. Il propose un cadre multi-critère qui se compose: (*i*) d'un modèle de réseau qui prend en compte le caractère diffusant du médium radio ainsi que la limitation due aux interférences sur le canal de transmission, (*ii*) d'une modélisation pour la dérivation de métriques de performance en régime permanent inspirée d'une chaine de Markov discrète et (*iii*) d'une formulation du problème d'optimisation multi-objectif qui permet d'obtenir des bornes supérieures multi-critères sur les performances du réseau. Les bornes Pareto-optimales qui présentent une borne supérieure sur les les compromis entre le délai de bout en bout et l'énergie consommée pour un réseau multi-saut à 1 et 2 relais sont données. Elles sont validées à l'aide de simulations.

Mots-clés : limites fondamentales, capacité, délai, énergie, réseau sans-fil, borne supérieure, borne multicritères

1 Introduction

Wireless ad hoc or sensor networks are many times operating in difficult environments and require several performance criteria to be satisfied, related to timely, reliable, and secure information transfer. Routing and resource allocation protocols are key solutions for ensuring proper information transfer across the network but their design has to cope with imperfect wireless transmissions. Finding the fundamental performance limits of wireless ad hoc networks is a very active research area (cf. [2] and references herein) that has triggered a comprehensive research effort in the last decade. Initial research has focused on defining the capacity region of the network [3, 6] and finding capacity maximizing resource allocation and routes [5].

Depending on the application, additional metrics such as delay, outage or energy consumption have become relevant. They are inter-dependent and can not be optimized separately. In this perspective, new multiobjective frameworks (network models and optimization tools) are needed to derive a multiobjective (MO) performance bound defined as the fundamental trade-off between the objectives of interest. These MO bounds could provide insight to improve network design and performance, as well as an upper bound against which to compare the performance of existing protocols [2].

The derivation of bounds relies on the definition of cross-layer network models capturing wireless variations on which a joint optimization of routing and resource allocation is performed [6, 5]. As proposed by Goldsmith et al. in [2], a promising way towards characterizing fundamental MO bounds in wireless ad hoc networks is to exploit "the broadcast features of wireless transmissions through generalized network coding, including cooperation and relaying". In this paper we propose a framework composed of a cross-layer network model and a steady-state performance evaluation model capturing capacity, delay and energy metrics. We formulate the associated MO optimization problem whose resolution provides both the MO bound and the MO optimal network configurations. Unique to this work is that our network model and MO problem has been designed to incorporate broadcast and interference-limited channels and thus, is capable of deriving bounds for a *layerless* communication paradigm [2] that integrates generalized network coding, cooperation and relaying.

The paper is organized as follows. The descriptions of our cross-layer network model and the relative MO optimization problem are given in Section 2 and 3, respectively. Section 4 presents our steady-state performance evaluation metrics. First results are presented in Section 5 and Section 6 concludes the paper.

2 Network model

We assume a synchronized network where transmissions are time-multiplexed. A frame of $|\mathcal{T}|$ time slots is repeated indefinitely. One or more packets (or symbols) can be transmitted in a time slot. In the rest of this paper, our examples assume that one packet is being sent in one time slot. A *time epoch* s is defined as the time needed to transmit one frame of $|\mathcal{T}|$ time slots. For any time slot $u \in \mathcal{T}$, there is an interference-limited channel between any two nodes i and j of the network. This channel is modeled by the probability of a symbol or a packet to be correctly transmitted between i and j in time slot u. This probability is referred to as the *channel probability* and denoted p_{ij}^u in the following. It models interference as an additive noise and is computed considering the distribution of the bit error rates (BER) or the packet error rates (PER) as shown hereafter.

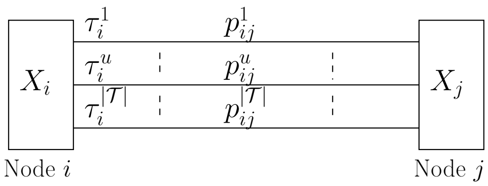

Figure 1: Network model

A wireless ad hoc network is modeled in this work by a finite weighted multiple edges complete graph $\mathcal{K}_{|\mathcal{V}|} = (\mathcal{V}, \mathcal{E})$ with $\mathcal{V}$ the set of vertices and $\mathcal{E}$ the set of edges. Two vertices are linked by $|\mathcal{T}|$ edges representing orthogonal interference-limited channels as illustrated on Fig. 1. In this graph, an edge (i, j, u) represents the channel between nodes i and j in time slot u. Each edge is assigned a weight of p_{ij}^u. For each node $i \in \mathcal{V}$, $\overrightarrow{\mathcal{N}_i}$ and $\overleftarrow{\mathcal{N}_i}$ are the set of edges leaving from and going into i, respectively. Each channel is assumed to be in a half-duplex mode, i.e. a node cannot transmit and receive a packet at the same time. An *emission* is defined as the couple $(i, u) \in \mathcal{V} \times \mathcal{T}$ and represents the fact that node i is emitting in a time slot u.

A set of sources $\mathcal{O}$ and destinations $\mathcal{D}$ is defined. A source $S_o \in \mathcal{O}, o \in \{1, .., |\mathcal{O}|\}$ can emit a flow of data to a single destination $D_d \in \mathcal{D}, d \in \{1, .., |\mathcal{D}|\}$ or to several ones. We make the assumption that source and destination nodes do not relay the information. Sources only emit packets and destinations only receive packets. Multi-hop transmissions are allowed and we model the other nodes as relay nodes $\mathcal{R} = \mathcal{V} - \mathcal{O} - \mathcal{D}$. We have $N = |\mathcal{R}|$ the number of relays in the network.

The network is synchronized. Depending on their time slot assignments, sources and relays emit their symbols (or packets) at the beginning of their assigned time slots. Nodes emitting in the same time slot may interfere, reducing reception probability of their emitted data. Nodes that are not emitting in a time slot can receive symbols (or packets) in this time slot. We assume that relays have an incoming buffer and $|\mathcal{T}|$ outgoing buffers. All buffers are able to store the amount of symbols (or packets) transmitted in one time slot duration. In our examples, they can store one packet. A relay receiving a packet has to decide if it will discard it or in which slot of the next frame it will send it. We consider as well in our model that a relay can not differentiate packets: identical packets are indiscernible.

2.1 Emission rate

A node j emits a flow of symbols (or packets) in time slot v. Symbols are the realization of a random variable $\overrightarrow{X_j^v}$ chosen from an alphabet $\mathcal{X}$ ($\mathcal{X} = \{0, 1\}$ for instance). In the graph, it means that node j emits the same flow on its

$\overrightarrow{\mathcal{N}_j^b}$ outgoing edges using the time slot v. Thus, our model incorporates the broadcast property of wireless communications.

A flow of symbols coming into node j from node i on time slot u is modeled as a random variable $\overleftarrow{Y_{ij}^u}$. The symbols are transmitted by node i on the edge $(i,j) \in \mathcal{E}$ in time slot u and consequently, experience the probability p_{ij}^u of being received successfully in j. Depending on the level of interference, symbols are received successfully or not at j.

Having this, let $\overleftarrow{Y_j} = \{\overleftarrow{Y_{ij}}, (i,j) \in \overleftarrow{\mathcal{N}_j}\}$ be the random variable giving all the symbols that can be received on all incoming edges of node j where $\overleftarrow{Y_{ij}} = \{\overleftarrow{Y_{ij}^u}, (i,j) \in \overleftarrow{\mathcal{N}_j^u}\}$. Let $\overrightarrow{X_j} = \{\overrightarrow{X_j^b}, j \in \overrightarrow{\mathcal{N}_j^b}\}$ be the random variable giving all the symbols that can be emitted on the outgoing channels of node j for all the time slots. The relation between the *outgoing* random variables $\overrightarrow{X_j}$ and the *incoming* random variables $\overleftarrow{Y_j}$ defines a coding scheme for the network.

In general, a relay has one main decision to take upon receiving a symbol or a packet: whether it should forward it or not. If it decides to forward it, the next step is to decide on which time slot to transmit it. If network coding is considered, the choice of the code adds to the set of decisions.

These decisions strongly influence the quality of the transmission and aim at mitigating the transmission errors due to fading and interference. Depending on these decisions, the emission rate of a node i on a time slot u varies. For instance, if a node decides to drop one packet out of two received on a same time slot u, the emission rate on channel u would become half the rate at which it received packets on the same time slot. Similarly, it is possible that a node transmits one packet every two received packets on time slot u because it is applying a coding scheme for which two packets are combined into a single transmitted packet through network coding. In both cases, the node is adapting the emission rate on each channel to combat errors.

Based on this statement, we characterize the behavior of a node i by the rate at which it is emitting its flow in each time slot u. The *emission rate* of node i in time slot u is denoted by τ_i^u. In our model, it is normalized and thus its value belongs to the interval $[0,1]$. The emission rate of destinations is equal to zero. Having this, a vector of emission rates for each time slot can be defined $\tau_i = \left[\begin{array}{ccc} \tau_i^1 & \dots & \tau_i^{|\mathcal{T}|} \end{array}\right]$. Emission rate τ_i^u can be interpreted as the probability node i is emitting packets in time slot u. Assume a time slot permits the transmission of one packet, a node i transmitting at a rate of $\tau_i^u = 0.5$ in time slot u will decide with probability 0.5 to transmit its packet in time slot u in the upcoming frame. If $\sum_{u\in\mathcal{T}} \tau_i = 0$, node i isn't active.

The average rate at which all the symbols are coming into node j is given by $\overleftarrow{r_j} = \sum_{u\in\mathcal{T}} \sum_{(i,j)\in\overrightarrow{\mathcal{N}_j^a}} \tau_i^u p_{ij}^u$ for p_{ij}^u the probability for the symbol to be correctly received. The rate at which symbols are being transmitted by node j is $\overrightarrow{r_j} = \sum_{v\in\mathcal{T}} \tau_j^v$.

Let $\tau = \left[\begin{array}{ccc} \tau_1^\dagger & \cdots & \tau_N^\dagger \end{array}\right]^\dagger$ be the *emission rate matrix.* A particular instance of τ belongs to the set of possible emission rate matrices denoted as Γ. An instance $\tau \in \Gamma$ is feasible if Properties 1 and 2 hold for each node:

Property 1: *Flow conservation.* The sum rate of all outgoing flows is lower or equal to the rate of all incoming flows, i.e.

$$\overrightarrow{r_j} \leq \overleftarrow{r_j}, \quad \forall j \in \mathcal{V} \tag{1}$$

PROPERTY 2: *Half duplex.* A node j is able to receive a message on a time slot u the proportion of time it is not transmitting on that same time slot. As a consequence, a τ is feasible if there is enough time to transmit and receive in any time slot for each node:

$$\overleftarrow{r_j^u} + \tau_j^u \leq 1, \quad \forall (j,u) \in \mathcal{V} \times \mathcal{T} \tag{2}$$

where $\overleftarrow{r_j^u} = \sum_{(i,j)\in \overrightarrow{\mathcal{N}_j^u}} \tau_i^u p_{ij}^u$ stands for the incoming cumulative rate in time slot u.

We define a set of *active emissions* $\mathcal{A}$ corresponding to the values of τ. An emission $(i,u) \in \mathcal{V} \times \mathcal{T}$ is said to be active if $\tau_i^u > 0$. As a consequence, the set $\mathcal{A}$ is given by $\mathcal{A} = \{(i,u) \in \mathcal{V} \times \mathcal{T} \mid \tau_i^u > 0\}$. Similarly, the set $\mathcal{A}^u$ refers to the set of active emissions restricted to time slot u. We have $\mathcal{A}^u = \{(i,v) \in \mathcal{V} \times \mathcal{T} | \tau_i^u > 0, v = u\}$

2.2 Channel probability

Knowing τ, it is possible to derive the interference level for each edge of the network. We want to stress here that the channel probability derivation is computed for a steady state where all nodes with $\tau_i^u \neq 0$ are transmitting concurrently in time slot u. We model interference as an additive noise computed at the end of each edge $(i,j,u) \in \mathcal{E}$ of the network. We recall that p_{ij}^u is the probability for a symbol emitted by node i to arrive successfully at node j in time slot u. It is a function of the statistical distribution of the Signal to Noise and Interference Ratio (SINR) at the location of the destination node j. This quantity can either be defined using a Packet Error Rate (PER) or a Bit Error Rate (BER) depending if transmissions on the network are packetized or not. In the following we consider that transmissions are packetized. Before giving the exact expression of the channel probability, a few preliminary definitions and notations are given hereafter:

Pathloss attenuation factor and transmission power a_{ij} reflects the attenuation due to propagation effects between nodes i and j for a simple isotropic model. All nodes use the same power P_T.

Interference Since transmissions are time-multiplexed, interference only occurs between transmissions using the same time slot. Let I_{ij}^u be the power of interference on the edge $(i,j,u) \in \mathcal{E}$ computed at node j. If $\mathcal{I}_{ij}^u$ is a set of nodes interfering the emission of i at node j in time slot u, interference power is calculated as $I_{ij}^u = \sum_{k\in\mathcal{I}_{ij}^u} P_T \cdot a_{kj}$.

SINR SINR between nodes i and j in time slot u is defined by $\gamma_{ij}^u = \frac{P_T \cdot a_{ij}}{N_0 + I_{ij}^u}$ where I_{ij}^u is the interference power on the link and N_0 the noise power density.

Packet error rate (PER) For a specific value of SINR γ, the packet error rate PER is computed with $PER(\gamma) = 1 - [1 - BER(\gamma)]^{N_b}$ where N_b is the number of bits of a data packet and $BER(\gamma)$ is the bit error rate for the specified SINR per bit γ which depends on the physical layer technology and the statistics of the channel. Results are given for an AWGN channel and a BPSK modulation without coding where $BER(\gamma) = 0.5 * \text{erfc}(\sqrt{\gamma})$.

We assume that there is no medium access mechanism and that emissions are totally independent. Within a time slot u, channel access is purely randomized and depends on the emission probabilities of the active nodes. Thus, on edge (i, j, u), the set of interferers present is random. In order to derive an average channel probability over time, we compute the average power of interference on the edge over time. Therefore we have to enumerate all possible configurations where active nodes interfere with the emitter i. These configurations are designated as interfering sets.

Interfering sets We recall that $\mathcal{A}^u$ is the set of active emissions in time slot u. An interfering set $\mathcal{I}_{ij}^u$ for edge (i, j, u) belongs to the set of all possible interfering sets $\mathcal{L}_{ij}^u$ on this edge. $\mathcal{L}_{ij}^u$ is the power set $\mathcal{P}(\mathcal{A}^u - \{i\})$, i.e. the set of all subsets of $\mathcal{A}^u - \{i\}$. Because of the half-duplex constraint, the receiver j is kept in the set $\mathcal{A}^u - \{i\}$. Thus, the channel probability computation accounts for the interfering sets where j is active. If j is active, the SINR is very low and transmission on the edge (i, j, u) is impossible.

Equation (3) details the derivation of the channel probability p_{ij}^u as the average of the PER experienced for all possible interfering sets $l \in \mathcal{L}_{ij}^u$ on edge (i, j, u) referred to as PER_l:

$$p_{ij}^u = \sum_{l \in \mathcal{L}_{ij}^u} [1 - PER_l] .\mathbf{P}_l \tag{3}$$

where $PER_l = PER(\gamma_l), l \in \mathcal{L}_{ij}^u$ and γ_l is the SINR experienced on the edge (i, j, u) when the nodes of the interfering set l are active.

$\mathbf{P}_l$ is the probability for nodes of the interfering set l to be active and create interference on (i, j, u). More specifically, it is the probability that the nodes of the interfering set l are transmitting concurrently and the others are not:

$$\mathbf{P}_l = \prod_{k \in l} \tau_k^u \cdot \prod_{m \in \{\mathcal{A}^u \setminus l\}} (1 - \tau_m^u) \tag{4}$$

In (4), $\prod_{k \in l} \tau_k^u$ gives the probability that the active nodes of the interfering set l are transmitting and the other product the probability that the $\{|\mathcal{A}^u \setminus l\}|$ other active nodes are not.

Similarly to the vector of emission rates, we define the vector of channel probabilities for a link $(i, j) \in \mathcal{E}$: $P_{ij} = \begin{bmatrix} p_{ij}^1 & \cdots & p_{ij}^{|\mathcal{T}|} \end{bmatrix}$. Let $P_j = \begin{bmatrix} P_{1j}^\dagger & \cdots & P_{Nj}^\dagger \end{bmatrix}^\dagger$ be the matrix giving all incoming channel probabilities at node j and $P = \begin{bmatrix} P_1 & \cdots & P_N \end{bmatrix}$ the matrix of all channel probabilities.

2.3 Forwarding and scheduling decisions

This section introduces forwarding and scheduling decisions of the nodes. These decisions are represented by the probability x_{ij}^{uv} of a node j to transmit on time slot v a packet coming from node i on time slot u. We will refer in the rest of the text to the forwarding probability x_{ij}^{uv}. For each node of the network, we can define a $N|\mathcal{T}|$-by-$|\mathcal{T}|$ matrix giving all the forwarding probabilities relative to any node j of the network as follows. This forwarding matrix is given by $X_j = \begin{bmatrix} X_{1j} & \cdots & X_{Nj} \end{bmatrix}^\dagger$ where each matrix X_{ij} provides the scheduling probabilities of a flow of packets coming from node i on its output times slots, depending on the time slot the packets are received on. We have

$$X_{ij} = \begin{bmatrix} x_{ij}^{11} & \cdots & x_{ij}^{1|\mathcal{T}|} \\ \vdots & & \vdots \\ x_{ij}^{|\mathcal{T}|1} & \cdots & x_{ij}^{|\mathcal{T}||\mathcal{T}|} \end{bmatrix}.$$

The matrix of forwarding probabilities is related to the matrix of emission rates τ and the matrix of channel probabilities P with the following set of $|\mathcal{A}|$ equations

$$\sum_{(i,j)\in\overrightarrow{\mathcal{N}_j^u}} \sum_{u\in\mathcal{T}} \tau_i^u p_{ij}^u x_{ij}^{uv} = \tau_j^v, \quad \forall(j,v)\in\mathcal{A} \tag{5}$$

where $\tau_i^u p_{ij}^u$ is the probability that a packet sent by i on time slot u arrives in j. These equations ensure a flow conservation. They strictly constrain the choices of forwarding probabilities.

The forwarding probabilities represent the decisions of the nodes to either (i) retransmit all the packets or symbols received or (ii) reduce the output rate by dropping or re-encoding them together. From now on, we will refer to the set of all forwarding probabilities of the complete network using a matrix $X = [X_1 \dots X_N]$, $X \in \mathcal{X}$ of size $N.|\mathcal{T}|$-by-$N.|\mathcal{T}|$ where $\mathcal{X}$ is the set of all possible matrix instances.

2.4 Illustrative examples

1-relay network In this example, we consider a basic 1-relay network as the one depicted in Fig. 2 ($N = 1$). Location of all nodes are known. For clarity

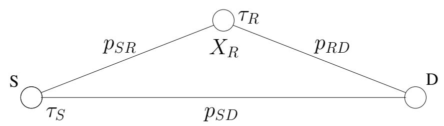

Figure 2: 1-Relay network

purposes, we illustrate the proposed network model with an interference free time slot allocation. A frame is composed of $|\mathcal{T}| = 2$ times slots, S emits in time slot 1 and relay R in time slot 2. As a consequence we have $\tau_S = \begin{bmatrix}\tau_S^1 & 0\end{bmatrix}$ and $\tau_R = \begin{bmatrix}0 & \tau_R^2\end{bmatrix}$. The forwarding probabilities matrix is:

$$X_R = \begin{bmatrix} x_{SR}^{11} & x_{SR}^{12} \\ x_{SR}^{21} & x_{SR}^{22} \end{bmatrix} = \begin{bmatrix} 0 & \tau_R^2/(\tau_S^1.p_{SR}^1) \\ 0 & 0 \end{bmatrix}$$

where the values of the x_{ij}^{uv} are derived according to the system of (5). From our model, we can for instance easily derive the average number of packets arriving at D which is the sum of the probabilities to receive a packet from all possible paths:

$$f = \tau_S^1.p_{SD}^1 + \tau_S^1.p_{SR}^1.x_{SR}^{12}.p_{RD}^2$$

2-relay network The 2-relay example is shown in Fig. 3. $|\mathcal{T}| = 3$ times slots

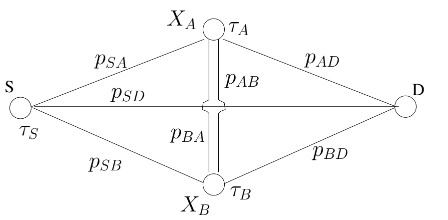

Figure 3: 2-Relay network

are considered for illutration. The source is still transmitting in time slot 1, relay A and B are transmitting in time slot 2 and 3, respectively. Thus, we have $\tau_S = \begin{bmatrix}\tau_S^1 & 0 & 0\end{bmatrix}$, $\tau_A = \begin{bmatrix}0 & \tau_A^2 & 0\end{bmatrix}$ and $\tau_B = \begin{bmatrix}0 & 0 & \tau_B^3\end{bmatrix}$. The only non-null forwarding probabilities are x_{AB}^{23}, x_{BA}^{32}, x_{SA}^{12} et x_{SB}^{13}. The system of (5) provides 2 constraints on the forwarding probabilities:

$$\begin{array}{c} \tau_S^1 . p_{SA}^1 . x_{SA}^{12} + \tau_B^3 . p_{BA}^3 . x_{BA}^{32} = \tau_A^2 \\ \tau_S^1 . p_{SB}^1 . x_{SB}^{13} + \tau_A^2 . p_{AB}^2 . x_{AB}^{23} = \tau_B^3 \end{array} \tag{6}$$

For a set of forwarding probabilities that respect the constraints in (6), the average number of copies f of a packet emitted by S arriving at D is computed by the infinite sum of the probabilities of receiving a packet through any possible path. In the two relay network, there is a loop between A and B which may create an infinite number of paths of increasing length. If $p_{AB}^2 x_{AB}^{23} p_{BA}^3 x_{BA}^{32} < 1$, f can be computed by:

$$\begin{array}{c} f = \tau_S^1 \Big[p_{SD}^1 + \frac{1}{1 - p_{AB}^2 x_{AB}^{23} p_{BA}^3 x_{BA}^{32}} \big(p_{SA}^1 x_{SA}^{12} p_{AD}^2 + \\ p_{SB}^1 x_{SB}^{13} p_{BD}^3 + p_{SA}^1 x_{SA}^{12} p_{AB}^2 x_{AB}^{23} p_{BD}^3 + \\ p_{SB}^1 x_{SB}^{13} p_{BA}^3 x_{BA}^{32} p_{AD}^2 \big) \Big] \end{array}$$

3 MO optimization problem

The scope of this section is now to take advantage of the previously described network model to derive a framework capable of extracting the set of Pareto-optimal networking solutions with respect to given performance criteria. A Pareto optimal set is composed of all the non-dominated solutions of the MO problem with respect to the performance metrics considered. The definition of dominance is:

Definition 1: A solution x dominates a solution y for a $n-$objective MO problem if X is at least as good as y for all the objectives and x is strictly better than y for at least one objective.

3.1 Solution and search space

Based on our network model, several parameters can be treated as optimization variables: the location of the N relays, the number of relays N, the number of time slots $|\mathcal{T}|$ and the forwarding probabilities represented by matrix $X \in \mathcal{X}$. In this paper, considered variables are the location of the N relays and their respective forwarding probabilities. Location of relays can be chosen in a convex set $\mathcal{C}$. For one network realization, the forwarding probabilities are chosen in the set $\mathcal{X}$. Thus the complete search space is $\mathcal{S} = \mathcal{C}^N \times \mathcal{X}$.

Let f_C, f_D and f_E be the performance criteria relative to capacity, delay and energy, respectively. Capacity is maximized, while energy and delay are minimized. The derivation of these metrics is provided in the next section. Our goal is to solve the following multiobjective optimization problem by finding the set of Pareto-optimal solutions $\mathcal{S}_{opt}$:

$$\mathcal{S}_{opt} = \{x \in \mathcal{S} \mid \forall x \in \mathcal{S}_{opt}, \forall y \in \mathcal{S}_{opt}^c, x \succ y\} \tag{7}$$

where $\mathcal{S}_{opt} \cup \mathcal{S}_{opt}^c = \mathcal{S}$ and notation $x \succ y$ means that x strictly dominates y. The cardinality of the search space $\mathcal{S}$ as defined grows exponentially with N,$|\mathcal{T}|$ and $|\mathcal{C}|$. There is a whole set of constraints for a solution $x \in \mathcal{S}$ to be valid already presented in (5). As a consequence, the solutions of $\mathcal{S}$ that do not follow these constraints are dropped before being evaluated by the search algorithm.

The question of choosing the optimal value N is out of the scope of this paper but will constitute an important extension of this work. This value is a trade-off between the search complexity and the quality of the Pareto-set obtained. Here, we consider small values for N. Similarly, results are presented for small values of $|\mathcal{T}|$ since we can achieve interference free transmission for $|\mathcal{T}| = N + 1$ time slot allocations.

3.2 Change of variable

The derivation of the transition rate matrix $\tau \in \Gamma$ knowing a solution $X \in \mathcal{X}$ is intractable. In a nutshell, to compute the emission rates of a node j, X and the incoming transmission probabilities P_j have to be known yet the elements of P_j are a function of the emission rates matrix, creating a circular dependency between the variables.

Nevertheless, being able to model exactly the interference level of the network is appealing and to keep the same formalism, we propose a reverse approach where a solution of the optimization problem is defined by the set τ of the emission rates for all the nodes and time slots. From τ, it is possible to derive the channel probabilities matrix P according to (4) since the activity of all nodes on each time slot is known.

Only instances of τ that meet the constraints relative to Property 1 and 2 are further considered as valid. Now that we have a valid τ, we can derive all the forwarding matrices $X \in \mathcal{X}$ that verify the constraints of equation (5). There are $|\mathcal{A}|$ constraints, each one constraining the choice of the x_{ij}^{uv} for all nodes and time slots of the network with respect to τ. Let $\mathcal{X}^\tau$ be the subset of $\mathcal{X}$ that verifies (5) with respect to the emission rate matrix τ. Each solution $X \in \mathcal{X}^\tau$ can be evaluated according to f_C, f_D and f_E. The MO optimization problem of (7) stays unchanged, however, the way the search space is constructed for one network configuration has changed and permits to select in a first stage a subset of valid solutions $\mathcal{X}^\tau$.

Table 1: Main notations in this section

$\mathcal{F}(s)$	Flow rate vector at time epoch s
$\overrightarrow{\mathcal{F}_i^u}(s)$	Outgoing flow rate vector after s time epochs
$\overleftarrow{\mathcal{F}_i^u}(s)$	Incoming flow rate vector after s time epochs
M	Transition matrix
Q	Relaying matrix
M_F	Fundamental matrix
M_S, D	Source and arrival matrix

4 Steady state evaluation

This section provides a framework for the definition of performance criteria for a particular solution $x \in \mathcal{S}$ of our MO optimization problem. It is an important contribution because it permits a fast performance derivation for end-to-end criteria such as capacity, delay and energy. The proposed derivation relies on the definition of a transition matrix which is composed of the probabilities for a flow of packets to be re-transmitted by the nodes of the network. Such a formulation is inspired by the theory of Markov chains. However, we want to stress that we do not model the network using a Markov chain. We define instead a transition matrix that has the properties needed to be able to re-use some results from the theory of Markov chains. Table 1 gives notations specific to this section.

4.1 Flow vectors

At time epoch 0, a source $S_o \in \mathcal{O}$ emits its flow of packets in a frame of $|\mathcal{T}|$ time slots following its emission rate vector. Here we make the assumption that the network is already fully loaded and that all relays and sources are active following the emission rate matrix τ.

After t time epochs, the flow of S_o has travelled in the network. All or part of it has reached different relays and because of losses on the channel, routing and scheduling decisions, its rate has most probably evolved. Let $\overrightarrow{\mathcal{F}_i^u}(s)$ represent the rate at which this flow is being emitted by node i in time slot u after s time epochs. Similarly, $\overleftarrow{\mathcal{F}_i^u}(s)$ represents the rate at which the same flow is being received at node i in time slot u from all its neighbors after s time epochs. For the rest of the paper, we will loosely designate $\overrightarrow{\mathcal{F}_i^u}(s)$ and $\overleftarrow{\mathcal{F}_i^u}(s)$ as outgoing and incoming flow rates respectively. The incoming flow rates are mostly of interest for destinations.

$\mathcal{F}(s)$ represents the vector of outgoing flow rates at the relays and the incoming flow rates of the destinations for each time slot after s time epochs. $\mathcal{F}(s)$ is referred to as the *flow rate vector* and is given by:

$$\mathcal{F}(s) = \left[\begin{array}{cc} \overrightarrow{\mathcal{F}_R}(s) & \overleftarrow{\mathcal{F}_D}(s) \end{array} \right]$$

where $\overrightarrow{\mathcal{F}_R}(s)$ stands for the outgoing flow rates emitted by the relays and $\overleftarrow{\mathcal{F}_D}(s)$ for the incoming flow rates received at the destinations after s time epochs. More specifically, we have:

$$\overrightarrow{\mathcal{F}_R}(s) = \left[\begin{array}{ccc} \overrightarrow{\mathcal{F}_1}(s) & \cdots & \overrightarrow{\mathcal{F}_N}(s) \end{array} \right]$$

with $\overrightarrow{\mathcal{F}_i}(s) = \left[\begin{array}{ccc} \overrightarrow{\mathcal{F}_i^1}(s) & \dots & \overrightarrow{\mathcal{F}_i^{|\mathcal{T}|}}(s) \end{array}\right]$ and

$$\overleftarrow{\mathcal{F}_D}(s) = \left[\begin{array}{c} \overleftarrow{\mathcal{F}_{D_1}}(s) \cdots \overleftarrow{\mathcal{F}_{D_{|\mathcal{D}|}}}(s) \end{array}\right]$$

with $\overleftarrow{\mathcal{F}_{D_d}}(s) = \left[\begin{array}{c} \overleftarrow{\mathcal{F}_{D_d}^1}(s) \cdots \overleftarrow{\mathcal{F}_{D_d}^{|\mathcal{T}|}}(s) \end{array}\right]$ the incoming flow rate received for each slot by each destination $D_d \in \mathcal{D}$.

The flow rate vector is of dimension $(N+|\mathcal{D}|)|\mathcal{T}|$ if all possible transmissions are accounted for. However, if node i never transmits in a time slot u (i.e. $\tau_i^u = 0$), $\overrightarrow{\mathcal{F}_i^u}(s) = 0$. As a consequence, the flow rate vector can be reduced to the size of the active emissions. Hence, the size of $\mathcal{F}(s)$ can be reduced to $(|\mathcal{A}| + |\mathcal{D}|)|\mathcal{T}|$.

4.2 Transition matrix

The propagation of the packets on the edges and the forwarding decisions of the nodes can be represented by a transition matrix M. When it is applied to the flow rate vector of time epoch s, the new flow rate vector of time epoch $(s+1)$ is obtained. Formally we have:

$$\mathcal{F}(s+1) = \mathcal{F}(s) \cdot M$$

The transition matrix M has the following structure:

$$M = \left[\begin{array}{c|c} Q & D \\ \hline 0 & I \end{array}\right]$$

For $l = N.|\mathcal{T}|$ and $m = |\mathcal{D}||\mathcal{T}|$, Q is a non-zero l-by-l matrix, D is a non-zero l-by-m matrix, I is a m-by-m identity matrix and 0 a m-by-l zero matrix. Q is referred to as the *relaying matrix* and D as the *arrival matrix*.

Transition matrix M has a similar canonical structure than a finite absorbing Markov chain [4]. Here, we have l transient states, i.e. relay nodes that are forwarding packets using active transmissions. The relaying matrix Q gives the probabilities for any emission (i,u) at time epoch s to be emitted as (j,v) at the following time epoch $(s+1)$ by the relays of the networks. We have m absorbing states given by the reception of packets at the destinations in $\mathcal{D}$. Identity matrix represents the fact that packets received by destinations are never forwarded, but absorbed. Matrix D is composed of the probabilities to go from any transient state to any absorbing state, i.e. the probabilities for any emission (i,u) at time epoch s to arrive at a destination D_d in time slot v at time epoch $(s+1)$.

The relaying matrix Q is structured as follows:

$$Q = \left[\begin{array}{cccc} 0 & Q_{12} & \cdots & Q_{1N} \\ Q_{21} & 0 & \cdots & Q_{2N} \\ \vdots & & & \vdots \\ Q_{N1} & \cdots & Q_{N-1N} & 0 \end{array}\right]$$

0 is an $|\mathcal{T}|$-by-$|\mathcal{T}|$ zero matrix representing the fact that node i never forwards a packet to itself. The matrix Q_{ij} is a $|\mathcal{T}|$-by-$|\mathcal{T}|$ matrix that gives the probabilities of node j to transmit a packet sent by node i for all possible combinations

of time slots:

$$Q_{ij} = \begin{bmatrix} Q_{ij}^{11} & \cdots & Q_{ij}^{1|\mathcal{T}|} \\ \vdots & & \vdots \\ Q_{ij}^{|\mathcal{T}|1} & \cdots & Q_{ij}^{|\mathcal{T}||\mathcal{T}|} \end{bmatrix} \tag{8}$$

where Q_{ij}^{uv} is the probability for a node j to retransmit on channel v a packet that has been transmitted by node i on time slot u. From our network model, it equals:

$$Q_{ij}^{uv} = p_{ij}^{u} x_{ij}^{uv}$$

The arrival matrix D is given by:

$$D = \begin{bmatrix} D_{1D_1} & \cdots & D_{1D_{|\mathcal{D}|}} \\ \vdots & & \vdots \\ D_{ND_1} & \cdots & D_{ND_{|\mathcal{D}|}} \end{bmatrix}$$

D_{iD_d} is a $|\mathcal{T}|$-by-$|\mathcal{T}|$ diagonal matrix whose diagonal elements $D_{iD_d}^{u}$ give the probabilities for a packet transmitted by a node i in time slot u to arrive at destination D_d. Consequently, $D_{iD_d}^{u} = p_{iD_d}^{u}$.

The source matrix M_S calculates the initial flow vector $\mathcal{F}(1)$ knowing the emission rate vector of any source in $\mathcal{O}$. An initial flow vector of source $S_o \in \mathcal{O}$ is obtained from M_S with:

$$\mathcal{F}(1) = \tau_{S_o} \cdot M_S$$

where M_S is a $|\mathcal{T}|$-by-$(N + |\mathcal{D}|).|\mathcal{T}|$ transition matrix defined as $M_S = [Q_S \ D_S]$ with Q_S and D_S the relaying and arrival matrix for the packets sent by the source, respectively. We have $Q_S = \begin{bmatrix} Q_{S_o1} \cdots Q_{S_oN} \end{bmatrix}$ and $D_S = \begin{bmatrix} D_{S_oD_1} \cdots D_{S_oD_{|\mathcal{D}|}} \end{bmatrix}$ where Q_{S_oi} follows the pattern given by (8) and $D_{S_oD_d}$ is a $|\mathcal{T}|$-by-$|\mathcal{T}|$ diagonal matrix whose diagonal elements are $D_{S_oD_d} = p_{S_oD_d}^{u}$.

The multiplication of τ_{S_o} with Q_S computes the outgoing flows at the relays for source S_o, i.e. it computes the probability for any relay to forward a packet directly received from S_o. The multiplication of τ_{S_o} with D_S computes the flows coming into the destinations for source S_o, i.e. it computes the probability for any destination to receive a packet directly from S_o.

The channel probability computation of (3) accounts for the interference distribution created by all sources and relays in the network. Thus, once the channel probabilities are set, it is possible to derive the outgoing flows for the packets of each individual source using the same source and transition matrices. If follows that M_S and Q can be applied to any type of source-destination communication (unicast, multicast, ...). This is a direct consequence of the fact that the inherent broadcast and interference-limited properties of the radio channel are accounted for in our network model. For instance, for a multicast transmission, the arrival matrix D is defined for $|\mathcal{D}| > 1$ destinations. The matrix Q stays unchanged compared to a unicast transmission with $|\mathcal{D}| = 1$. In a convergecast scenario where multiple sources are transmitting to a same destination (a sink for instance in a wireless sensor network), each source is propagated in the network using the same Q to a unique destination modeled by D. An important consequence for our optimization problem, is that its complexity is not a function of the number of flows transiting in the network.

In the following, matrices are represented considering all N relays. However, when evaluating a specific solution τ, these matrices are derived only for the set $\mathcal{A}$ of active emissions and the size of the square matrix Q is reduced from $l = N.|\mathcal{T}|$ to $l = |\mathcal{A}|$.

4.3 Fundamental matrix

We really want to stress that the proposed transition matrix does not follow the definition of a regular Markov chain. Indeed, the sum of the probabilities on a line of M is greater than 1. However, if Q is convergent, Q^s tends to zero as s tends to infinity and the transmission reaches a steady state. In this case, the following theorem of absorbing Markov chains still holds:

THEOREM 1: $I - Q$ has an inverse, and

$$(I-Q)^{-1} = I + Q + Q^2 + \cdots = \sum_{s=0}^{\infty} Q^s$$

Proof A complete proof is omitted for conciseness purposes but main intuition is given. For Theorem 1 to hold, Q must be convergent. In absorbing Markov chain theory, this is the case if from any non-absorbing state it is possible to reach an absorbing state. An element of Q gives the probability of a node to re-emit a packet transmitted by another node. Due to the broadcast property, a packet can be re-emitted by multiple nodes, introducing non-null elements in the relay flow vector $\overrightarrow{\mathcal{F}_R}(s)$ as the number of time epochs (and hops) increases. Thus, any created packet (original or copy) has to reach a destination node to be absorbed. This is true for any packet traveling on a path *without loop* since it is being absorbed by a destination.

Packets traveling on paths *with loops* may infinitely circle in the loop and never reach a destination. We can prove that for the elementary loop present in the 2-relay network, packets are absorbed if the maximum eigenvalue of Q is strictly lower than one. This is the case if $Q_{AB}^{23} \cdot Q_{BA}^{32} < 1$, i.e. $p_{AB}^2 x_{AB}^{23} p_{BA}^3 x_{BA}^{32} < 1$ which is the probability for a packet entering the loop at A to be received at A again. For a larger loop, it can be shown by induction that if the probability of a packet entering the loop at a node i to be received at i (which we designate as the loop probability) is higher than or equal to one, Q is not convergent. Thus, Q is convergent if all paths with loops have a loop probability strictly lower than one. ■

Practically, solutions $x \in \mathcal{S}$ with non convergent Q can be disregarded from the MO optimization search since they lead to solutions creating an infinite number of received packets, which is detrimental to energy and delay minimization.

The inverse of $I - Q$ is defined as the *fundamental matrix* and denoted $M_F = (I - Q)^{-1}$. An element ${M_F}_{ij}^{uv}$ of the fundamental matrix represents the average number of re-emissions node j has done in v from packets sent by i in u until all packets are received at the destinations.

Knowing M_F, it is possible to derive from $\mathcal{F}(1)$ the *steady state flow vector* $\mathcal{F}(\infty) = \left[\overrightarrow{\mathcal{F}_R}(\infty), \overleftarrow{\mathcal{F}_D}(\infty)\right]$:

$$\overrightarrow{\mathcal{F}_R}(\infty) = \overrightarrow{\mathcal{F}_R}(1) \cdot M_F$$

$$\overleftarrow{\mathcal{F}_D}(\infty) = \left[\begin{array}{cc} \overrightarrow{\mathcal{F}_R}(\infty) & \overleftarrow{\mathcal{F}_D}(1) \end{array} \right] \cdot \left[\begin{array}{c} D \\ I \end{array} \right]$$

where I is a $|\mathcal{D}||\mathcal{T}|$-by-$|\mathcal{D}||\mathcal{T}|$ identity matrix.

The steady state flows transmitted by the relays $\overrightarrow{\mathcal{F}_R}(\infty)$ are directly obtained from the fundamental matrix. The steady state flows coming into the destinations $\overleftarrow{\mathcal{F}_D}(\infty)$ are obtained in two steps: propagating the steady state relay flows to the destination using D and adding to it the direct transmissions from the source to the destinations $\overleftarrow{\mathcal{F}_D}(1)$ of time epoch 1. An element of $\overleftarrow{\mathcal{F}^u_{D_d}}(\infty)$ gives the steady state sum of all packets being received at destination D_d in time slot u.

4.4 Optimization criteria

The definition of our optimization criteria are directly derived from the fundamental matrix. Indeed, if $I - Q$ is invertible, M_F gives a measure of the performance of one solution $x \in \mathcal{D}$ when the number of time epochs tends to infinity. The very interesting feature is that it also accounts for all the possible cycles in the graph if Q is convergent and consequently models the broadcast property of the wireless channel. The criteria here after are defined for one source-destination flow. In the following, we consider source $S_o \in \mathcal{O}$ transmits to destination $D_d \in \mathcal{D}$. Since the underlying network model holds for concurrent flows, it is possible to easily define criteria for multiple flows in the future.

Redundancy, capacity and reliability The number of packets absorbed at the destinations for one packet emitted by S_o is given by:

$$f = \overleftarrow{\mathcal{F}_D}(\infty) \cdot I_D(d).$$

$I_D(d)$ is a selection vector of dimension $|\mathcal{D}||\mathcal{T}|$ where the $|\mathcal{T}|$ elements relative to destination D_d are equal to 1 and the others are equal to 0. This vector accumulates the packets received in each time slot at destination D_d. This criterion means that, with network solution $x \in \mathcal{D}$, f packets are received at D_d for one packet sent by S_o. However, since relays are not able to discriminate packets, the set of f packets may be composed in the worst case of f copies of a packet sent by the source. What we can say here is that:

• if $f \geq 1$, there may be at most f different packets. Consequently, f provides an upper bound on both network capacity and packet redundancy. Let f_C be the bound on the real capacity of the network:

$$f_C = \min(1, f)$$

This bound is exact if there exists a practical transmission scheme where all packets received at the destination are different.

• if $f < 1$, f can be interpreted as a reliability criterion defined as the probability to get one packet at D_d.

Delay criterion Let $P(H = h)$ be the probability for a transmission towards D_d to be done in h hops. After s time epochs, a packet can travel up to $h = s+1$

hops. Thus

$$P(H=h)=\begin{cases}\overleftarrow{\mathcal{F}_D}(1)\cdot I_D(d) & h=1\\ \overrightarrow{\mathcal{F}_R}(1)\cdot Q^{h-2}\cdot D\cdot I_D(d) & h\geq 2\end{cases}$$

We assume that a relay introduces a delay of 1 unit. Consequently, a h-hop transmission introduces a delay of $h-1$ units. The average end to end delay is computed by the infinite sum:

$$f_D=\sum_{h=1}^{\infty}(h-1)\cdot P(H=h)=0\cdot\overleftarrow{\mathcal{F}_D}(1)+$$
$$\overrightarrow{\mathcal{F}_R}(1)\cdot\left[I+2Q+\cdots+(h+1)Q^h+\cdots\right]\cdot D\cdot I_D(d)$$

Let V be equal to $(I+2Q+\cdots+(h+1)Q^h+\cdots)$. We can show that $V=(M_F)^2$ and consequently

$$f_D=\overrightarrow{\mathcal{F}_R}(1)\cdot(M_F)^2\cdot D\cdot I_D(d)$$

Proof Infinite sum $\sum_{i=0}^{\infty} i\cdot Q^i$ is equal to $\frac{Q}{(1-Q)^2}$. This sum is equal to $Q\cdot V$. Thus, $V=\frac{1}{(1-Q)^2}$. By definition of $M_F=(1-Q)^{-1}$, $V=(M_F)^2$. ■

Energy criterion The energy is measured by the number of packets being emitted in the network by the source and the relays. By definition of the fundamental matrix, it is

$$f_E=\sum_{t=1}^{|\mathcal{T}|}\tau_o+\overrightarrow{\mathcal{F}_R}(1)\cdot M_F\cdot\left[\begin{array}{ccc}1 & \cdots & 1\end{array}\right]^{\dagger}$$

where $\left[\begin{array}{ccc}1 & \cdots & 1\end{array}\right]$ is a vector of $N|\mathcal{T}|$ ones that accumulates all the packets sent by the relays that have participated in the transmission. The first sum of the equation counts the number of packets sent by the source S_o.

4.5 Illustrative examples

1-Relay network We still consider the example of Fig. 2. The initial flow vector $\mathcal{F}(1)$ is decomposed as $\overrightarrow{\mathcal{F}_R}(1)=\left[\begin{array}{cc}0 & \tau_S^1.Q_{SR}^{12}\end{array}\right]$ and $\overleftarrow{\mathcal{F}_D}(1)=\left[\begin{array}{cc}\tau_S^1.D_{SD}^1 & 0\end{array}\right]$. Since $N=1$, $M_F=I$ and $\overrightarrow{\mathcal{F}_R}(\infty)=\overrightarrow{\mathcal{F}_R}(1)$.

By definition, $f=\overrightarrow{\mathcal{F}_R}(\infty).\left[\begin{array}{c}1\\1\end{array}\right]=\tau_S^1p_{SD}^1+\tau_S^1p_{SR}^1x_{SR}^{12}p_{RD}^2$. Criterion $f_D=\tau_S^1.p_{SR}^1.x_{SR}^{12}.p_{RD}^2$ since

$$f_D=\overrightarrow{\mathcal{F}_R}(\infty)\cdot\left[\begin{array}{cc}0 & 0\\0 & D_{RD}^2\end{array}\right]\cdot\left[\begin{array}{c}1\\1\end{array}\right]$$

and $f_E=\tau_S^1+\tau_S^1.p_{SR}^1.x_{SR}^{12}$ since

$$f_E=\tau_S^1+\overrightarrow{\mathcal{F}_R}(\infty)\cdot\left[\begin{array}{c}1\\1\end{array}\right].$$

2-relay network The 2-relay network of Fig. 3 is considered again. The source is transmitting in time slot 1, relay A and B are transmitting in time slot 2 and 3, respectively. The only non-null forwarding probabilities are x^{23}_{AB}, x^{32}_{BA}, x^{12}_{SA} et x^{13}_{SB}. Initial flow vector $\mathcal{F}(1)$ is composed of $\overrightarrow{\mathcal{F}_R}(1) = \tau^1_S \begin{bmatrix} Q^{12}_{SA} & Q^{13}_{SB} \end{bmatrix}$ and $\overleftarrow{\mathcal{F}_D}(1) = \begin{bmatrix} \tau_S D^1_{SD} & 0 & 0 \end{bmatrix}$. Q and M_F are equal to:

$$Q = \begin{bmatrix} 0 & Q^{23}_{AB} \\ Q^{32}_{BA} & 0 \end{bmatrix}; M_F = \frac{1}{|I-Q|} \begin{bmatrix} 1 & Q^{23}_{AB} \\ Q^{32}_{BA} & 1 \end{bmatrix}$$

with $|I-Q| = 1 - Q^{32}_{BA}.Q^{23}_{AB}$. Steady state outgoing relay flows are:

$$\overrightarrow{\mathcal{F}_R}(\infty) = \frac{\tau^1_S}{|I-Q|} \begin{bmatrix} Q^{12}_{SA} + Q^{13}_{SB}.Q^{32}_{BA} & Q^{13}_{SB} + Q^{12}_{SA}.Q^{23}_{AB} \end{bmatrix}$$

and the arrival matrix is $D = \begin{bmatrix} 0 & D^2_{AD} & 0 \\ 0 & 0 & D^3_{BD} \end{bmatrix}$.

The three criteria are:

$$f = G + \frac{\tau^1_S}{|I-Q|}.(E+F)$$

with $E = (Q^{12}_{SA} + Q^{13}_{SB}.Q^{32}_{BA}).D^2_{AD}$, $G = \tau_S D^1_{SD}$ and $F = (Q^{13}_{SB} + Q^{12}_{SA}.Q^{23}_{AB}).D^3_{BD}$.

$$f_D = \frac{\tau^1_S}{|I-Q|^2}(A+B)$$

with $A = D^2_{AD}[Q^{12}_{SA}(1 + Q^{23}_{AB}.Q^{32}_{BA}) + 2Q^{13}_{SB}.Q^{32}_{BA}]$ and $B = D^3_{BD}[Q^{13}_{SB}(1 + Q^{23}_{AB}.Q^{32}_{BA}) + 2Q^{12}_{SA}.Q^{23}_{AB}]$.

$$f_E = \tau^1_S + \frac{\tau^1_S}{|I-Q|}(Q^{12}_{SA} + Q^{13}_{SB}.Q^{32}_{BA} + Q^{12}_{SA}.Q^{23}_{AB} + Q^{13}_{SB}).$$

5 Results

In this section, we look for the Pareto bounds that concurrently minimize capacity-achieving delay f^*_D and energy f^*_E criteria for the 1-relay and 2-relay problems. Here, capacity achieving criteria f^*_D and f^*_E are obtained by dividing the value of f_D and f_E by $\min(f_C, 1)$ respectively. It permits to incorporate the effect of limited capacity on delay and energy in our analysis. For example, if $f = 0.5$, $f^*_D = 2f_D$ and $f^*_E = 2f_E$ which means that 2 times more packets have to be sent in average to reach perfect capacity which incurs double delay and energy. Theoretical Pareto-optimal solutions and bounds are obtained using non-dominated sorting genetic algorithm (NSGA-2) [1]. To assess our network model, we simulate the Pareto-optimal solutions using the event-driven network simulator WSNet[1].

5.1 Simulation settings

The distance between S and D is 620 meters. For $P_T = 0.151$mW and a pathloss exponent of 3, direct transmission probability without interference is

[1]Worldsens simulator, http://wsnet.gforge.inria.fr/

Table 2: RMSE for f_D^* and f_E^*

Cases	f_D^*	f_E^*
1-Relay case	8.4e-006	6.9e-006
2-Relay case	1.6e-004	5.7e-005

near 0. The set of Pareto optimal locations of relays is searched in a continuous rectangular surface area of size 620*620 square meters located in-between S and D. For NSGA-2, we use a population of size 500 and 1000 generations. The crossover probability is set to 0.9.

Each Pareto optimal solution of the MO bound is simulated with WSNet. In our simulations, a perfect TDMA is implemented following 1-relay and 2-relay problem specifications. S sends a packet every first time slot of every frame. Experiment is run for 10000 frames. For each Pareto optimal solution, the location of the relays and their forwarding probabilities are known. These forwarding probabilities are used in simulation as the decision to broadcast a packet upon its reception.

Values of f_D^* and f_E^* are calculated from simulated values of f, f_D and f_E derived as follows. f is measured by the total number of packets N_{rx} received at D (including copies) divided by the number of packets transmitted by S. f_D is the average transmission duration of the packets received at D. It is measured using the statistical distribution of the delays of the packets arrived over each possible distance measured in hops: $P(h) = n(h)/N_{rx}$, where $n(h)$ is the number of packets arrived in h hops at D. f_D is then calculated with $f_D = \sum_{h=1}^{\infty}(h-1).P(h)$. In this calculation, we consider as in our model that a transmission on a path of h hops introduces a delay of $h-1$ units. f_E is the sum of the number of packets transmitted by the source and the relays.

Error between our model and simulations is measured for each criterion with a normalized root-mean-square error (RMSE) computed as

$$RMSE = \frac{1}{N_{opt}}\sqrt{\sum_{i=1}^{N}\frac{(f_{model}(i) - f_{sim}(i))^2}{f_{model}(i)^2}}$$

where f_{model} and f_{sim} are the model and simulated values for criterion f and N_{opt} is the total number of Pareto-optimal solutions in the MO bound.

5.2 Simulation results

Table 2 gives the values of RMSE for both 1-relay and 2-relay problems. Values are really small, showing a quasi-perfect match between the model and simulations. Fig. 4 plots the delay-energy Pareto bounds for both 1-relay and 2-relay problems. Delay results of 1-relay and 2-relay cases have been divided by $|\mathcal{T}| = 2$ and $|\mathcal{T}| = 3$ respectively to be comparable. A clear compromise is visible: decreased energy is obtained at the price of increased delay. Less reliable solutions spend less energy but need more transmissions to achieve perfect capacity.

For the 1-relay case, Pareto-optimal relays are located in a small circle of radius 10^{-1}m located around the middle of $[S, D]$ and have a forwarding probability $x_{SR}^{12} = 1$. Variability of optimal criteria is small ($\simeq 8e10^{-2}$), meaning that the most important variable is here the location of R.

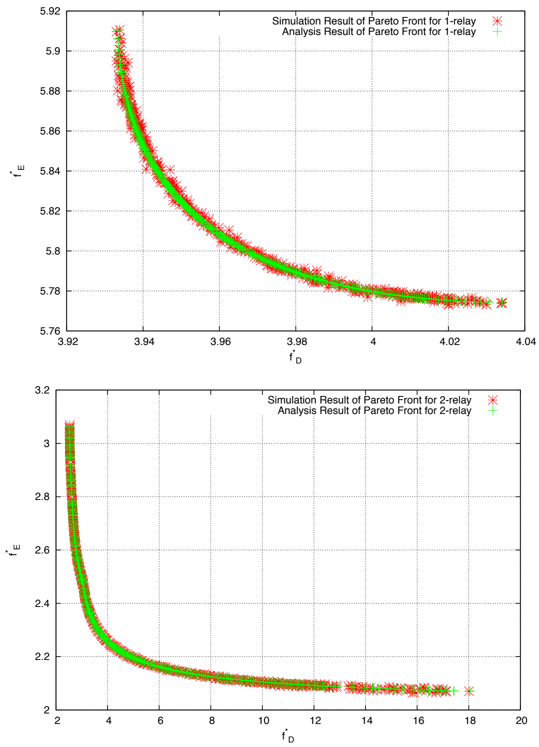

Figure 4: 1- (top) and 2-relay (bottom) bounds.

The 2-relay bound dominates the 1-relay bound, thus transmission is more efficient using 2-relay paths than 1-relay paths. It is a consequence of the path diversity introduced by the 2 relays. Solutions with minimal delay have relays close to each other in the center of $[S, D]$ and no loop exists ($x^{23}_{AB} = 0$ OR $x^{23}_{AB} = 0$). On the opposite, high delay solutions are located close together as well, but source-relay links are weaker, reducing end to end transmission probability.

6 Conclusion

The work presented in this paper has derived a flexible framework for evaluating the performance of a wireless ad hoc network with respect to several performance criteria. It has been designed to account for the broadcast nature of wireless communications and for an accurate interference characterization for the network. Our framework allows for the determination of both Pareto bounds and corresponding Pareto solutions. Network model and bounds for 1-relay and 2-relay networks have been assessed though simulations. The derived MO problem relies on an extended search space whose complexity mainly grows with N. Further works will consider proper choice for N and improved optimization approaches that better leverage the problem structure. However, compared to similar studies in mesh networks [5], the complexity of the proposed framework is scalable with respect to the number of flows in the network thanks to our definition of forwarding probabilities and our interference model.

The tightness (or pessimism) of the upper MO bound presented in this paper will be assessed in future works. Therefore, we are currently investigating combined fountain and network coding strategies to provide feasible transmission strategies which constitute lower MO bounds on performance.

use section* for acknowledgement

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